

Geometric Folding Algorithms: Bridging Theory to Practice

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Final report

1 Results

Research followed two main thrusts:

1.1 Reconfigurable Robots

- Solved the hinged dissection problem, which was over a 100 years old, proving that any finite collection of shapes have a hinged dissection.
- Proved that crystalline robots can reconfigure extremely efficiently: $O(\log n)$ time and $O(n)$ moves.
- Proved that any orthogonal polyhedron can be folded from a single, universal crease pattern (box pleating).

1.2 Origami Design

- Developed mathematical theory for what happens in paper between creases, in particular for the case of circular creases.
- Circular crease origami on permanent exhibition at MoMA in New York.
- Developing mathematical theory of Tomohiro Tachi's Origamizer framework for efficiently folding any polyhedron from a sheet of paper.
- Developing mathematical theory of Robert Lang's TreeMaker framework for efficiently folding tree-shaped origami "bases".
- Developing tools with Tomohiro Tachi for animation of real origami.
- Visited Japan in December 2007, sharing latest results in computational origami.

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2 Publications

2.1 Book

“Geometric Folding Algorithms: Linkages, Origami, Polyhedra” (joint work with Joseph O’Rourke), Cambridge University Press, July 2007.

2.2 Journal Papers

“Refolding Planar Polygons” (joint work with Hayley N. Iben and James F. O’Brien), *Discrete & Computational Geometry*, to appear. Special issue of selected papers from SoCG 2006.

“Grid Vertex-Unfolding Orthostacks” (joint work with John Iacono and Stefan Langerman), *International Journal of Computational Geometry and Applications*, to appear.

“Staged Self-Assembly: Nanomanufacture of Arbitrary Shapes with $O(1)$ Glues” (joint work with Martin L. Demaine, Sandor P. Fekete, Mashhood Ishaque, Eynat Rafalin, Robert T. Schweller, and Diane L. Souvaine), *Natural Computing*, volume 7, number 3, pages 347–370, Sept. 2008. Special issue of selected papers from DNA 2007.

“Edge-Unfolding Nested Polyhedral Bands” (joint work with Greg Aloupis, Stefan Langerman, Pat Morin, Joseph O’Rourke, Ileana Streinu, and Godfried Toussaint), *Computational Geometry: Theory and Applications*, volume 39, number 1, pages 30–42, Jan. 2008.

2.3 Book Chapters

“All Polygons Flip Finitely... Right?” (joint work with Blaise Gassend, Joseph O’Rourke, and Godfried T. Toussaint), in *Surveys on Discrete and Computational Geometry: Twenty Years Later*, J. Goodman, J. Pach, and R. Pollack, eds., Contemporary Mathematics 453, pages 231–255, 2008, American Mathematical Society.

2.4 Conference Papers

“Reconfiguration of Cube-Style Modular Robots Using $O(\log n)$ Parallel Moves” (joint work with Greg Aloupis, Sebastien Collette, Stefan Langerman, Vera Sacristan, and Stefanie Wuhler), in *Proceedings of the 19th Annual International Symposium on Algorithms and Computation*, to appear, Dec. 2008.

“Hinged Dissections Exist” (joint work with Timothy G. Abbott, Zachary Abel, David Charlton, Martin L. Demaine, and Scott D. Kominers), in *Proceedings of the 24th Annual ACM Symposium on Computational Geometry*, pages 110–119, College Park, MD, June 2008.

“Linear Reconfiguration of Cube-Style Modular Robots” (joint work with Greg Aloupis, Sebastien Collette, Mirela Damian, Robin Flatland, Stefan Langerman, Joseph O’Rourke, Suneeta Ramaswami, Vera Sacristan, and Stefanie Wuhler), in *Proceedings of the 18th Annual International Symposium on Algorithms and Computation*, pages 208–219, Dec. 2007.

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
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1. REPORT DATE (DD-MM-YYYY) 11-03-2009		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 07/15/2007 - 07/14/2008	
4. TITLE AND SUBTITLE Geometric Folding Algorithms: Bridging Theory to Practice			5a. CONTRACT NUMBER FA9550-07-1-0538		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) Erik Demaine			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Massachusetts Institute of Technology Computer Science and Artificial Intelligence Laboratory 32 Vassar St., Cambridge, MA 02139, USA			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NL 875 N RANDOLPH STREET SUITE 325, RM 3112 ARLINGTON VA 22203-1768			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-DSR-VA-TR-2012-0495		
12. DISTRIBUTION/AVAILABILITY STATEMENT Approve For Public Release					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT I. RECONFIGURABLE ROBOTS * Solved the hinged dissection problem, which was over a 100 years old, proving that any finite collection of shapes have a hinged dissection. * Proved that crystalline robots can reconfigure extremely efficiently: $O(\log n)$ time and $O(n)$ moves. * Proved that any orthogonal polyhedron can be folded from a single, universal crease pattern (box pleating). II. ORIGAMI DESIGN * Developed mathematical theory for what happens in paper between creases, in particular for the case of circular creases. * Circular crease origami on permanent exhibition at MoMA in New York. * Developing mathematical theory of Tomohiro Tachi's Origamizer framework for efficiently folding any polyhedron from a sheet of paper. * Developing mathematical theory of Robert Lang's TreeMaker framework for efficiently folding tree-shaped origami "bases".					
15. SUBJECT TERMS folding, geometry, algorithms, mathematics					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Erik Demaine
					19b. TELEPHONE NUMBER (Include area code) 617-253-6871

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